MTH 304: Metric Spaces and Topology

Homework IV

(Due 13/02)

1. Establish assertion 1.8 (iv) of the lesson plan.
2. Establish assertion 1.8 (v) of the lesson plan.
3. For an indexed family $\{X_\alpha\}_{\alpha\in J}$ of topological spaces, show that the projection map

$$\pi_\alpha : \prod_{\alpha\in J} X_\alpha \to X_\alpha$$

   is continuous in both the product and box topologies.

4. Let $\mathbb{R}_0^\infty \subset \mathbb{R}^\infty$ be defined by

$$\mathbb{R}_0^\infty = \{(x_n) \in \mathbb{R}^\infty \,|\, x_i \neq 0 \text{ only for finitely many i}\},$$

   that is the set of all sequences in $\mathbb{R}$ that are *eventually zero*. Find $\overline{\mathbb{R}_0^\infty}$ in $\mathbb{R}^\infty$ in both the product and box topologies.

5. Consider a map $T : \mathbb{R}^\infty \to \mathbb{R}^\infty$ that is linear in each coordinate.
   (a) Show that if $\mathbb{R}^\infty$ is given the product topology, then T is a homeomorphism.
   (b) Does (a) hold true if $\mathbb{R}^\infty$ is given the box topology?